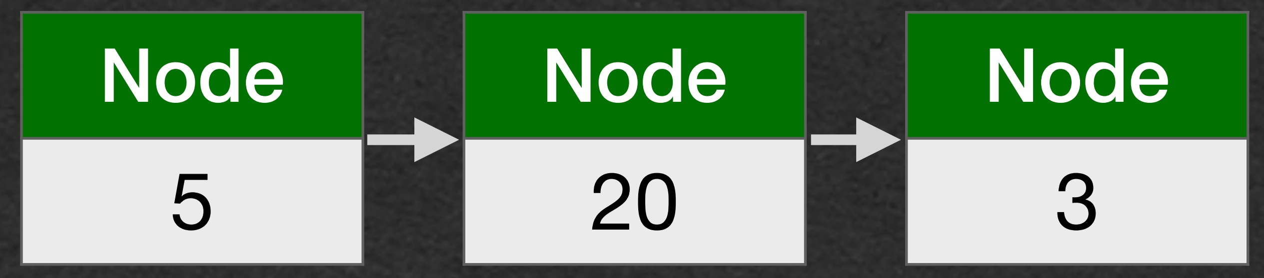
Graphs

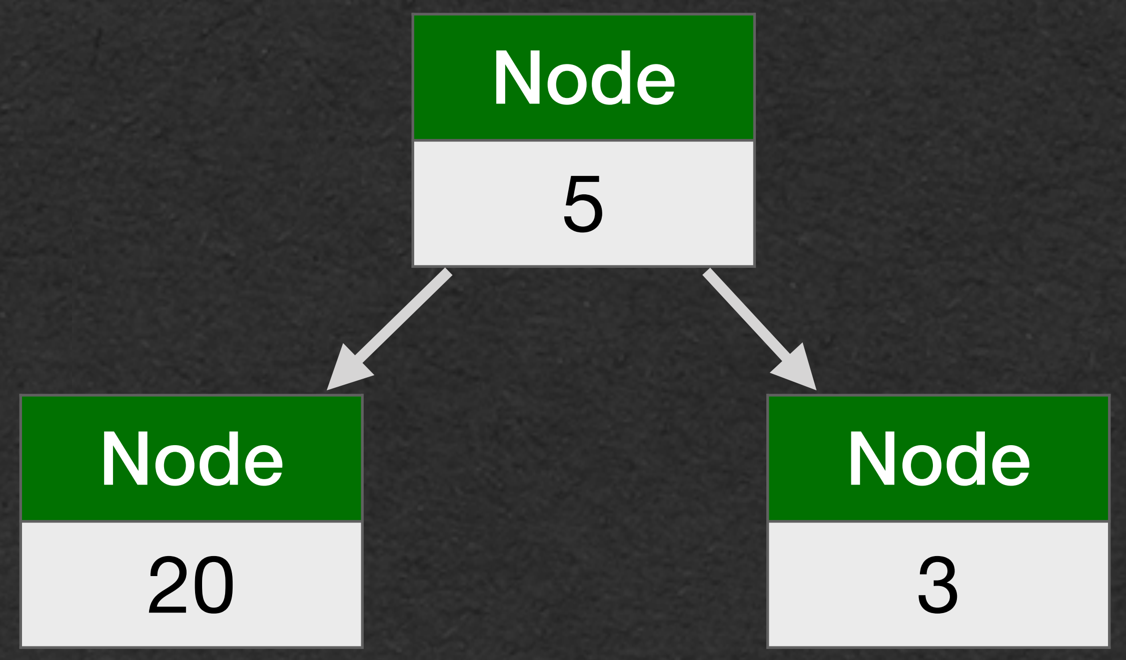
Data Structures: Review

- Sequential Data Structures
  - Elements stored in a specific order
  - Ex: Array, List
- Key-Value Store
  - Stores pairs of elements with no particular order
  - Each key is associated with one value
  - Ex. Map, Dictionary, Object
- Tree
  - Non-linear structure
  - Each element can be associated with multiple other elements

Index	0	1	2
Value	5	20	3

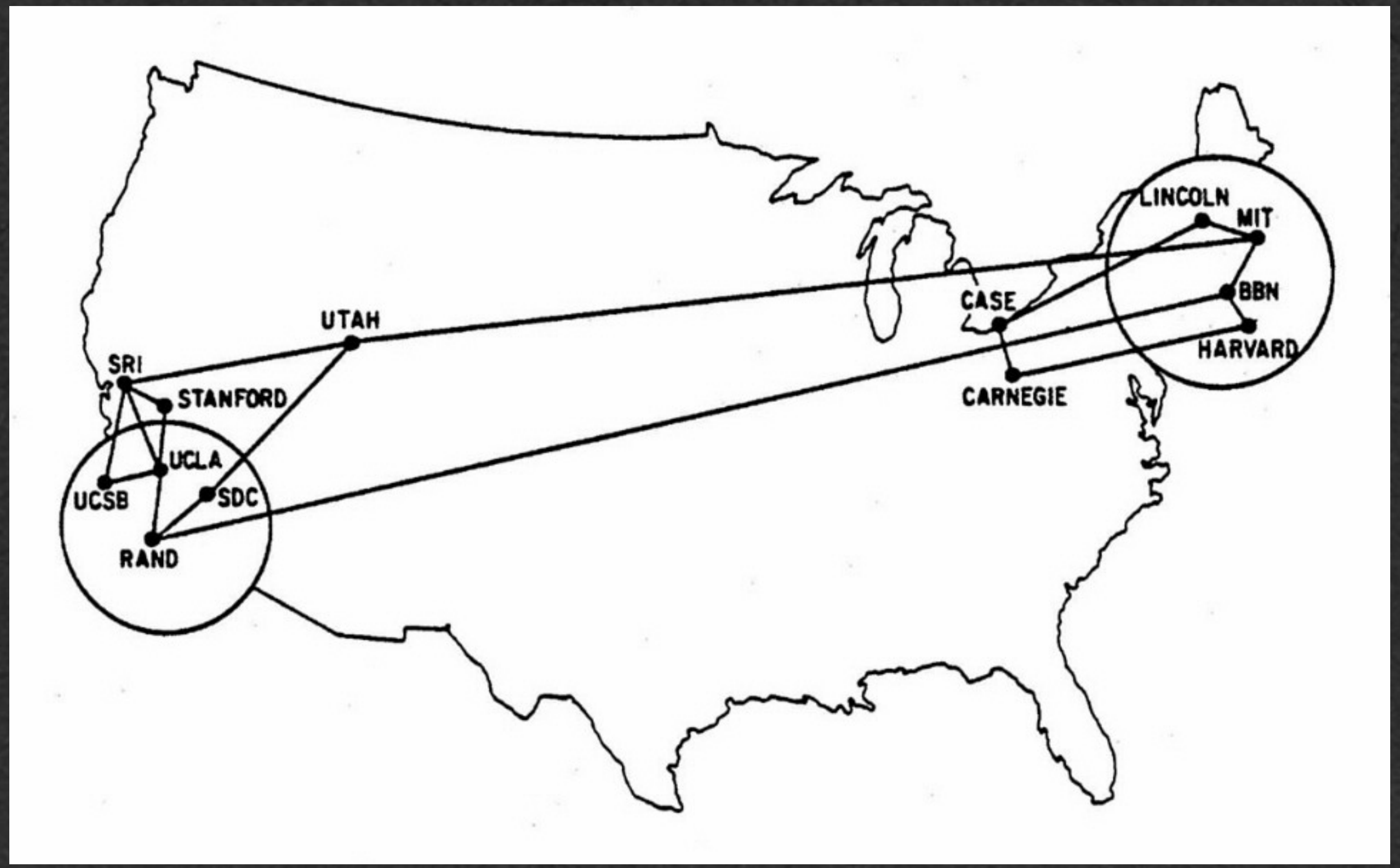


Key	"cse"	"mga"	"geo"
Value	20	3	5



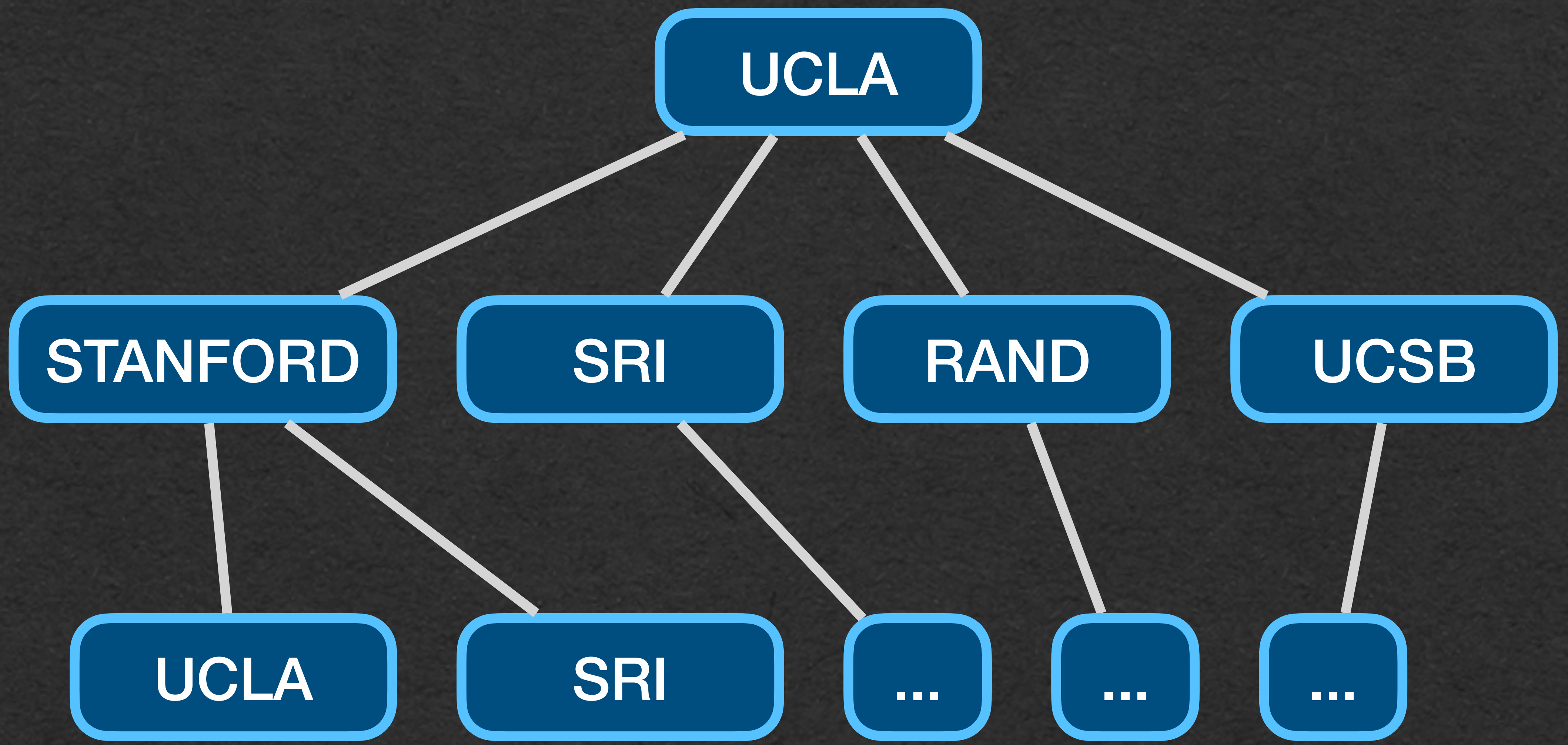
Data Structures

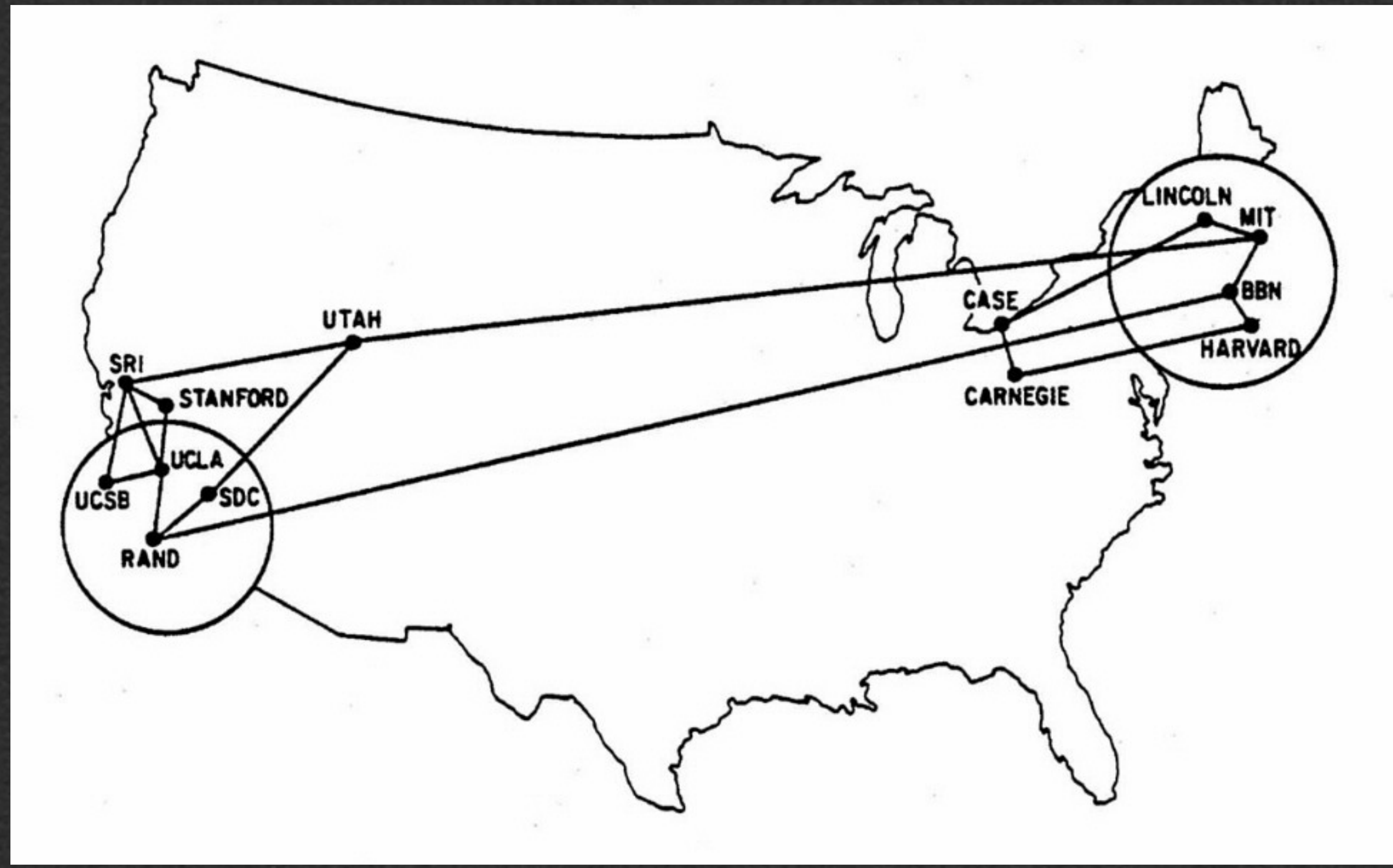
- How do we store data with multiple interconnected associations?
- A [station, intersection, city] can have multiple connections



Data Structures

- Let's use trees
- Start with UCLA as the root
- Recursively add all children

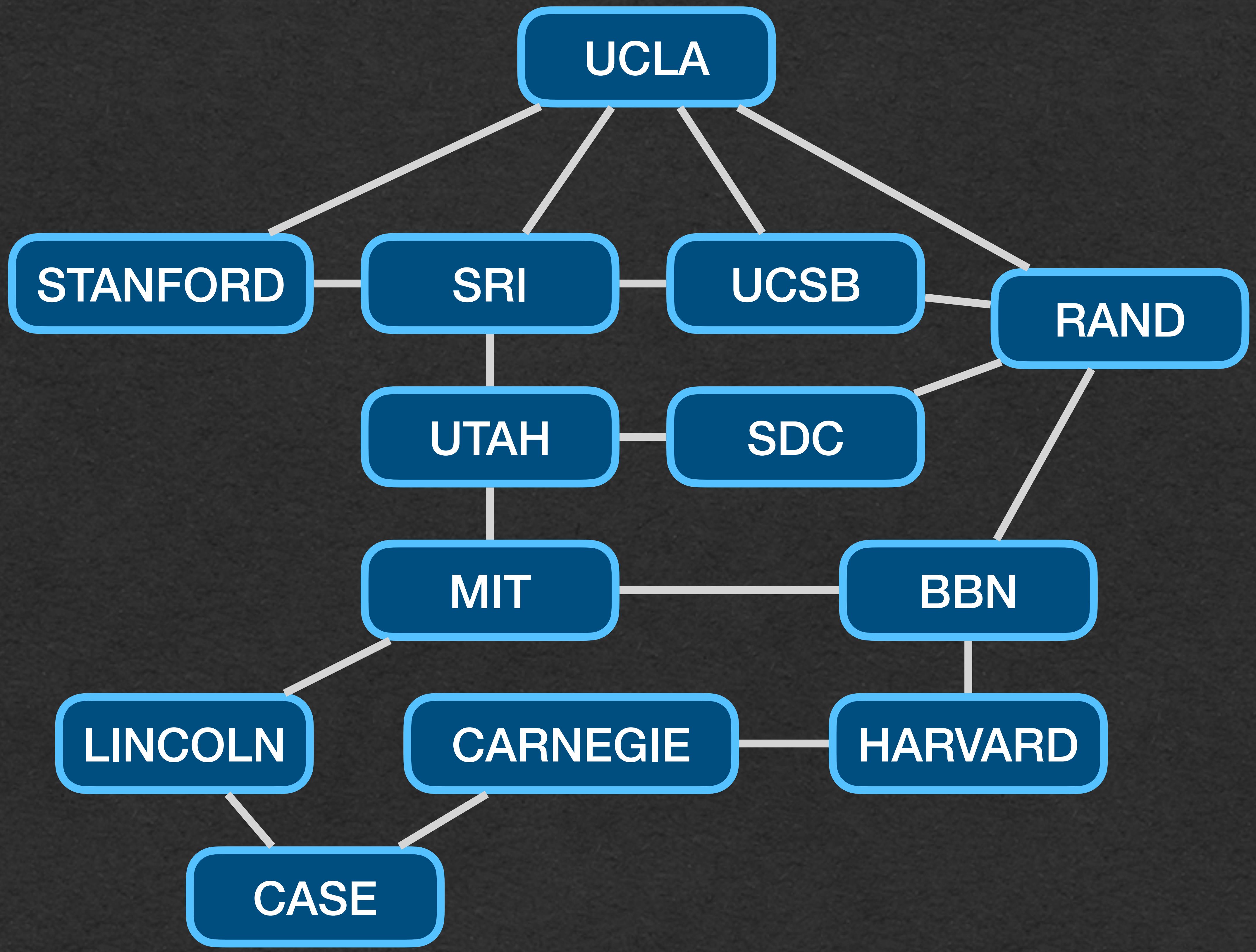


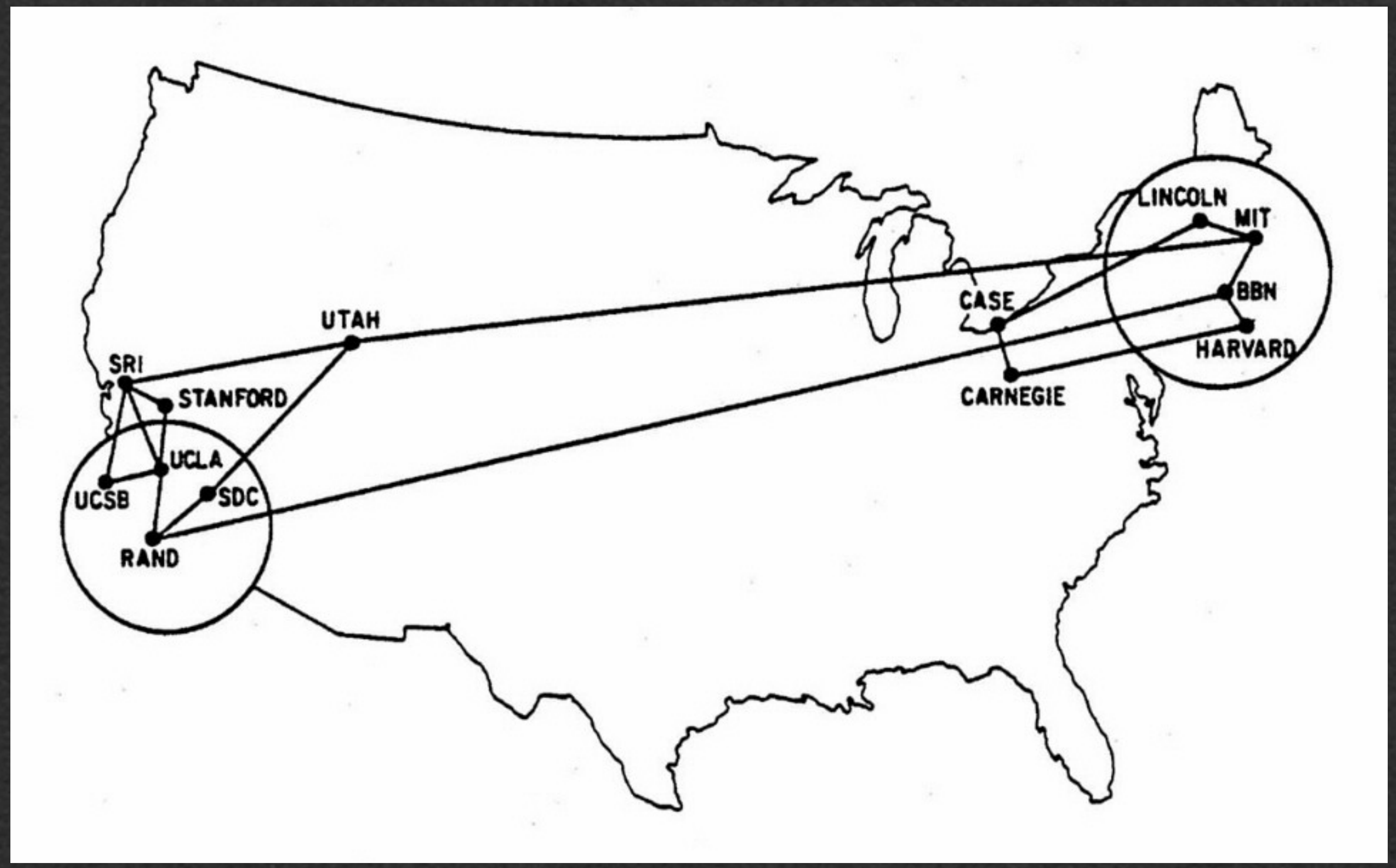


- Oops
    - We have duplicates in our data structure

Data Structures

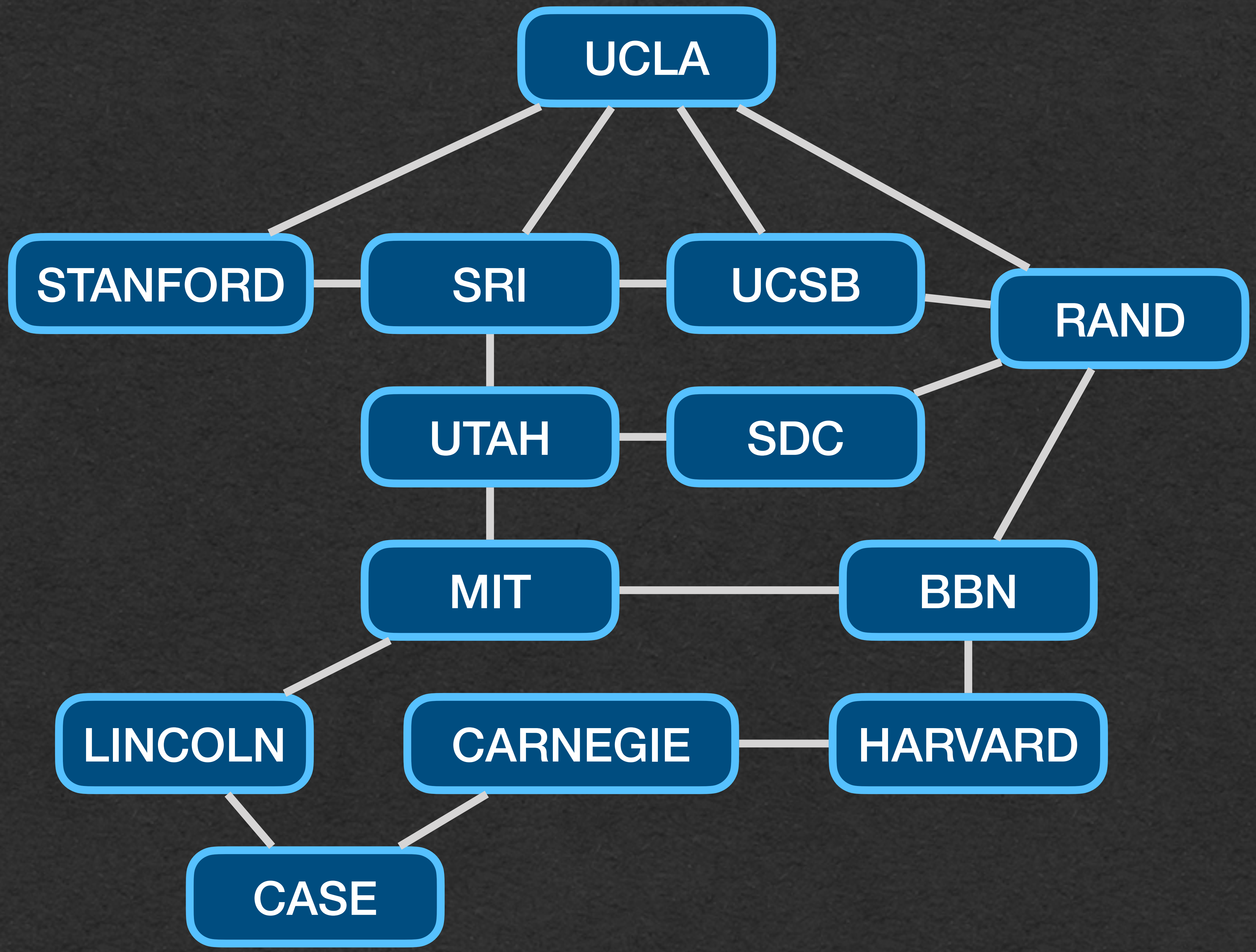
- Let's try again
  - When we try to add a duplicate, add a reference to the existing node

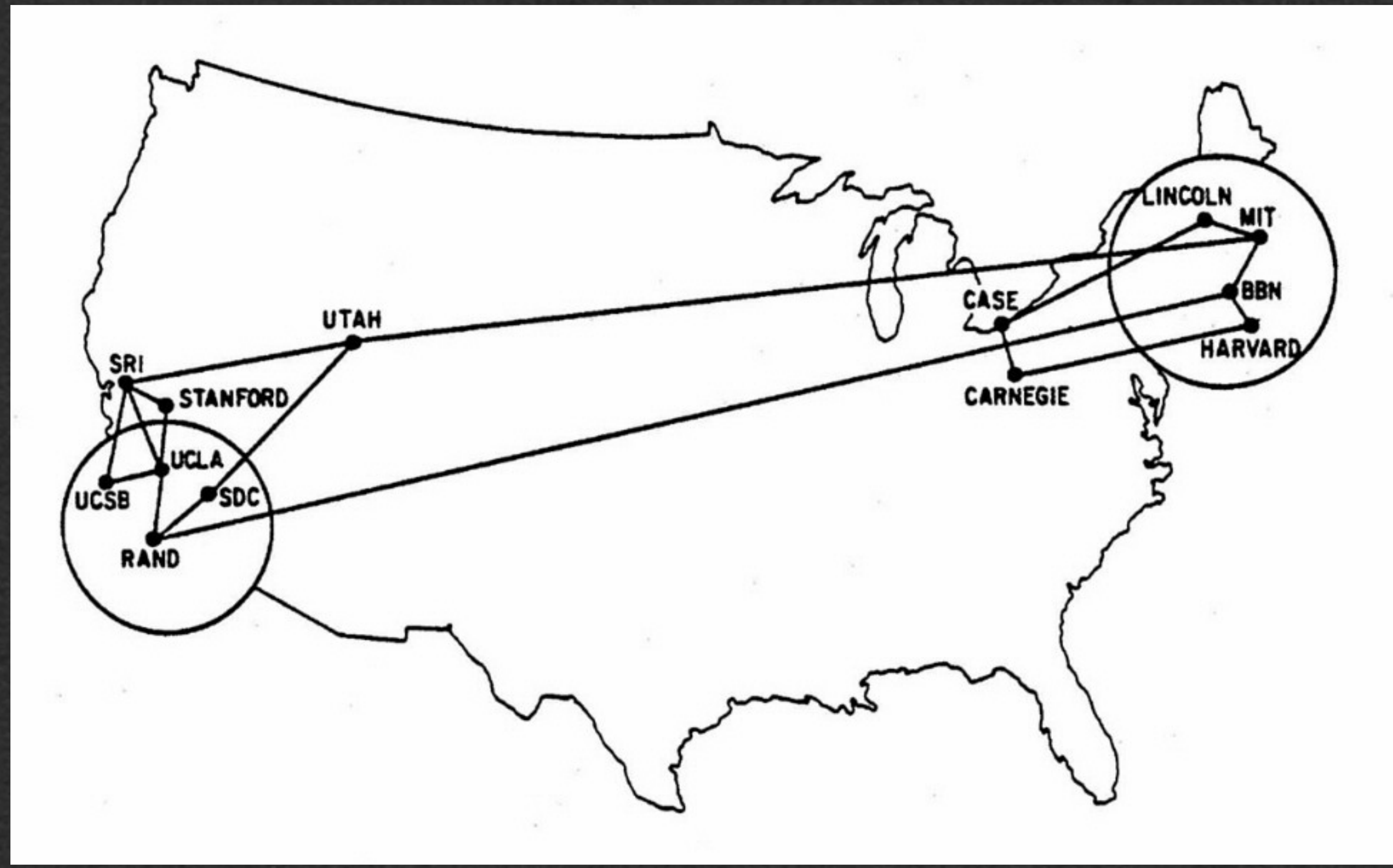




Graphs

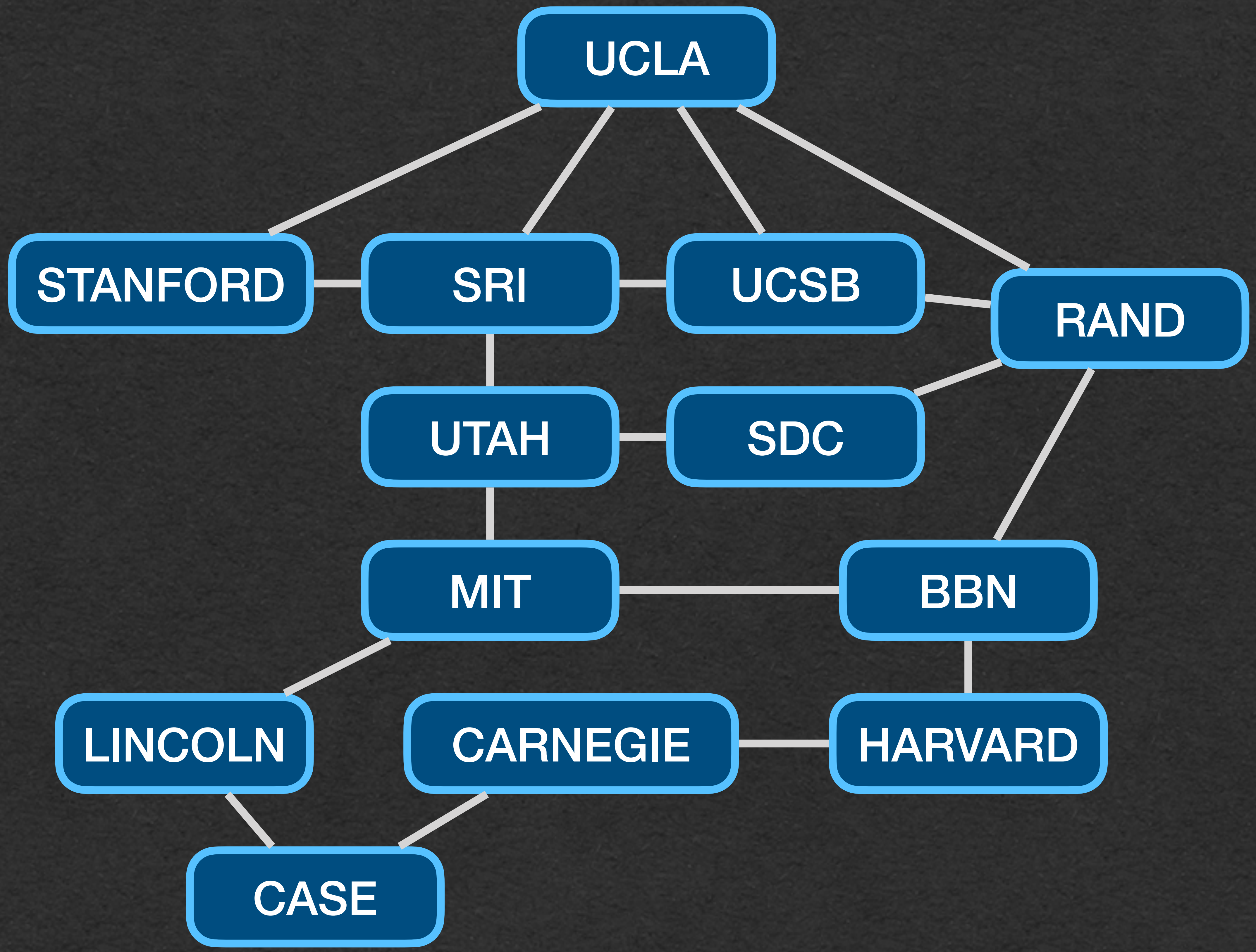
- This is a graph
- Similar to a tree, except cycles are allowed
    - Cycle: Can "travel" from a node back to itself without repeating a node

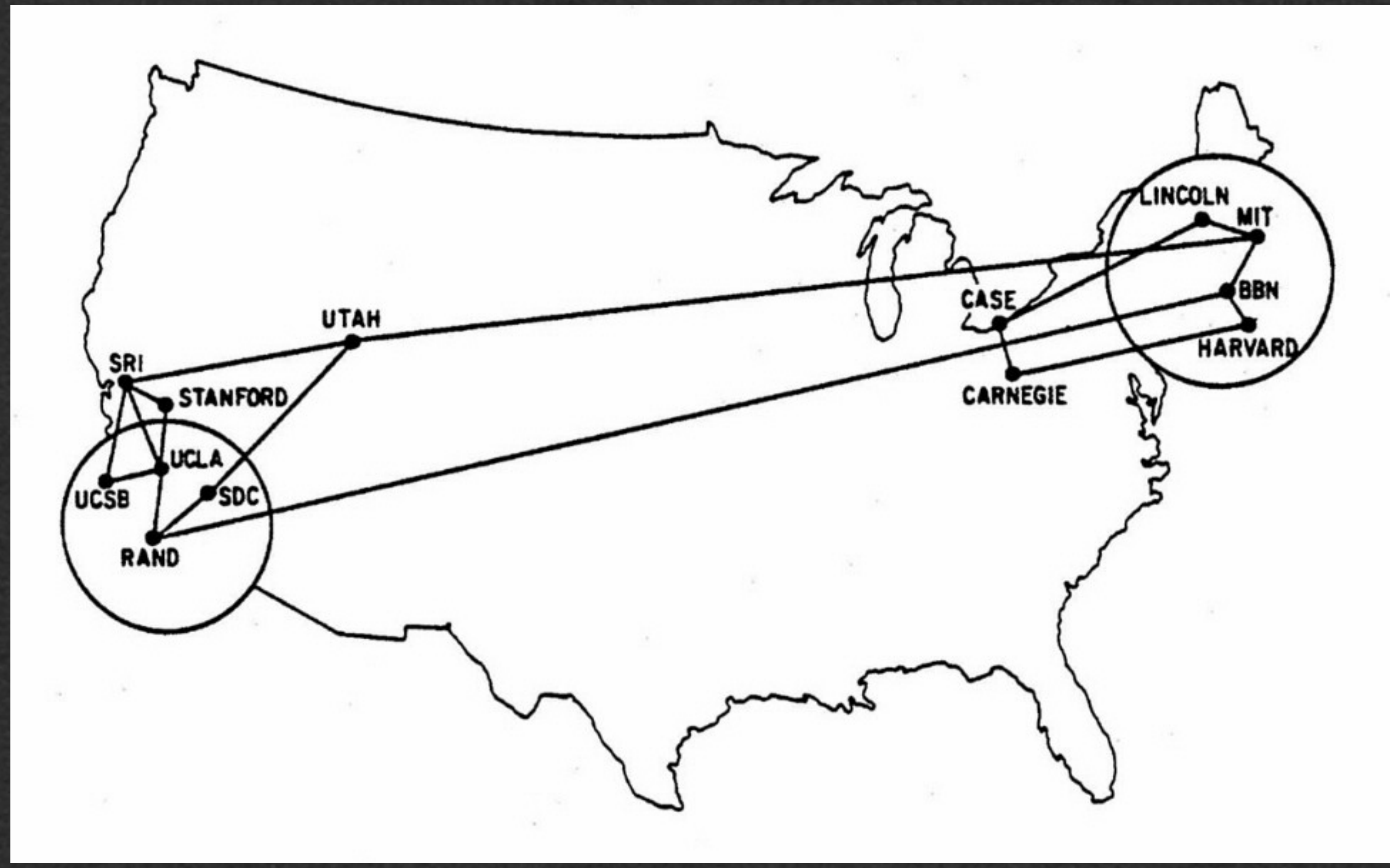




Graphs

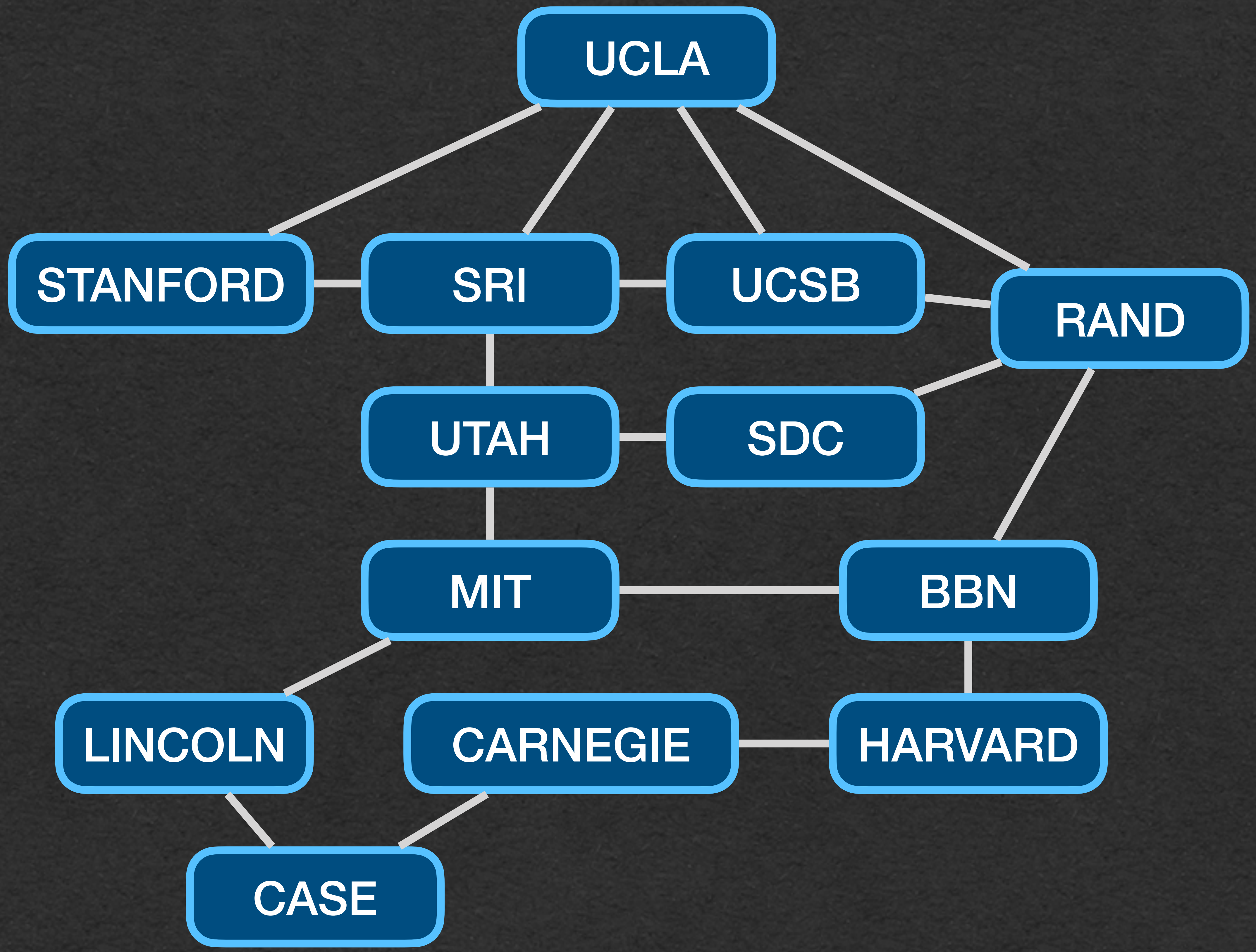
- Because of the cycles, our tree traversals will get stuck in infinite recursion
  - No leaves (node with no children) to terminate the recursion

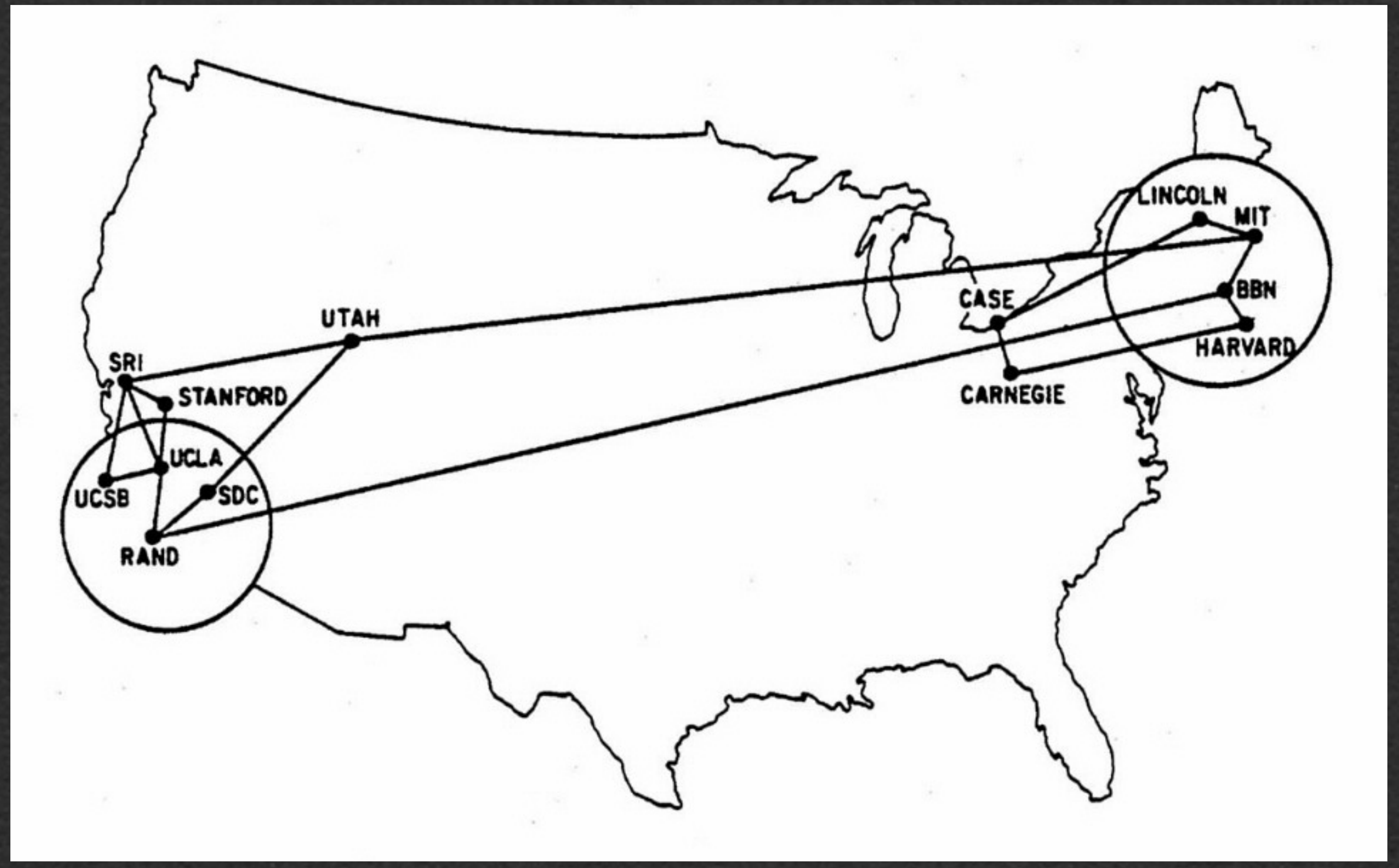




Graphs

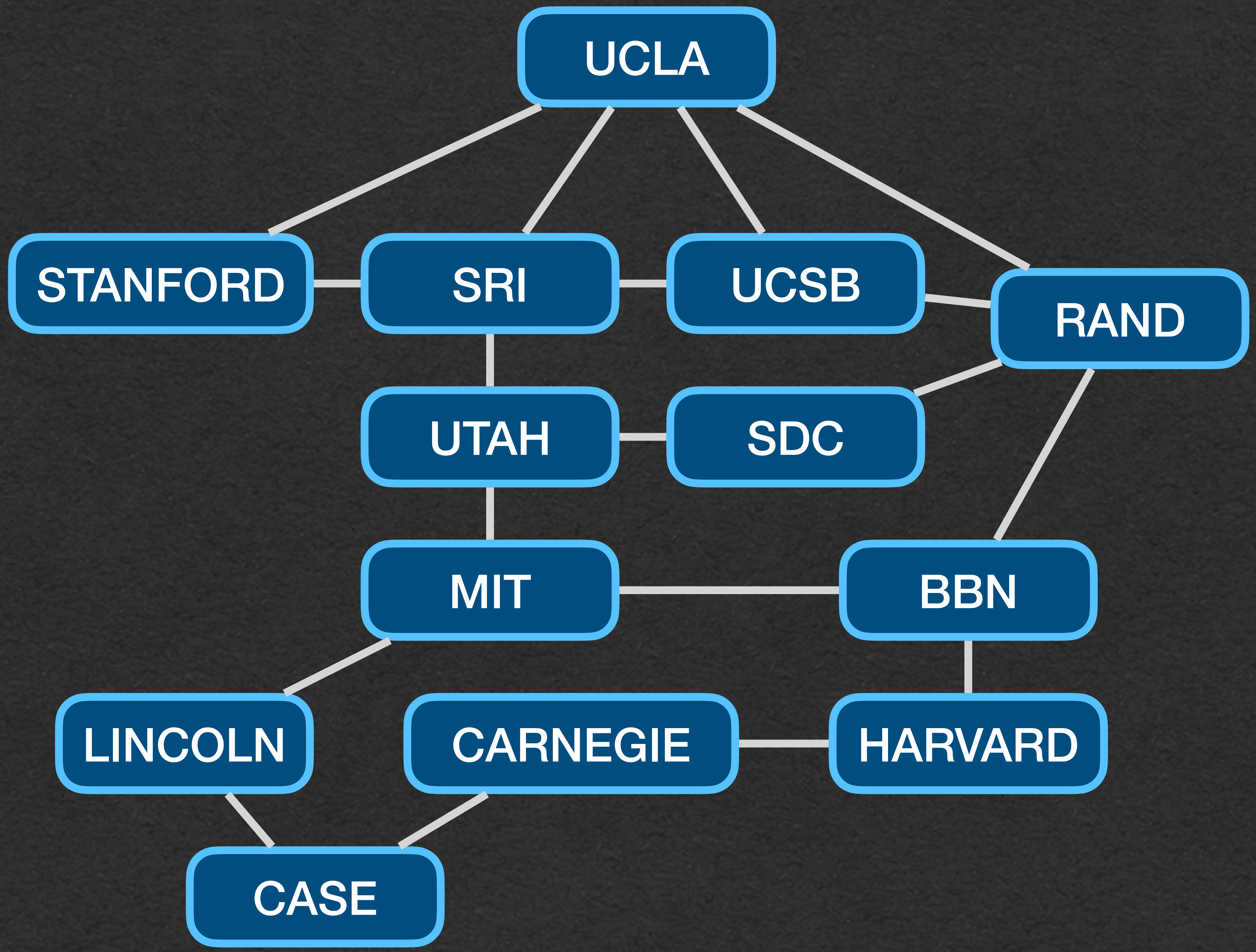
- We'll need a new way of representing this data structure and new algorithms to work with the data
- Store the nodes and edges

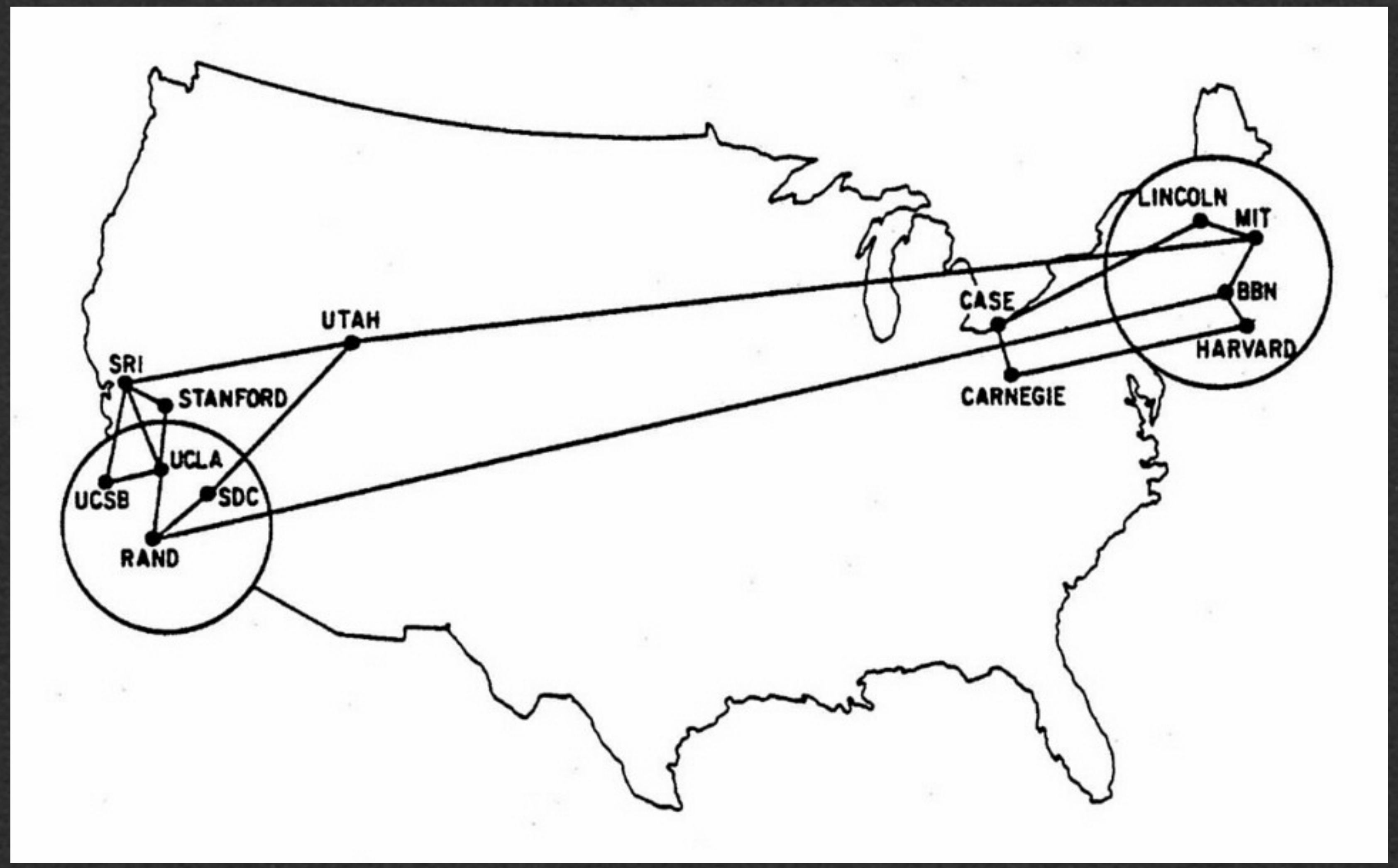




Graphs - Nodes and Edges

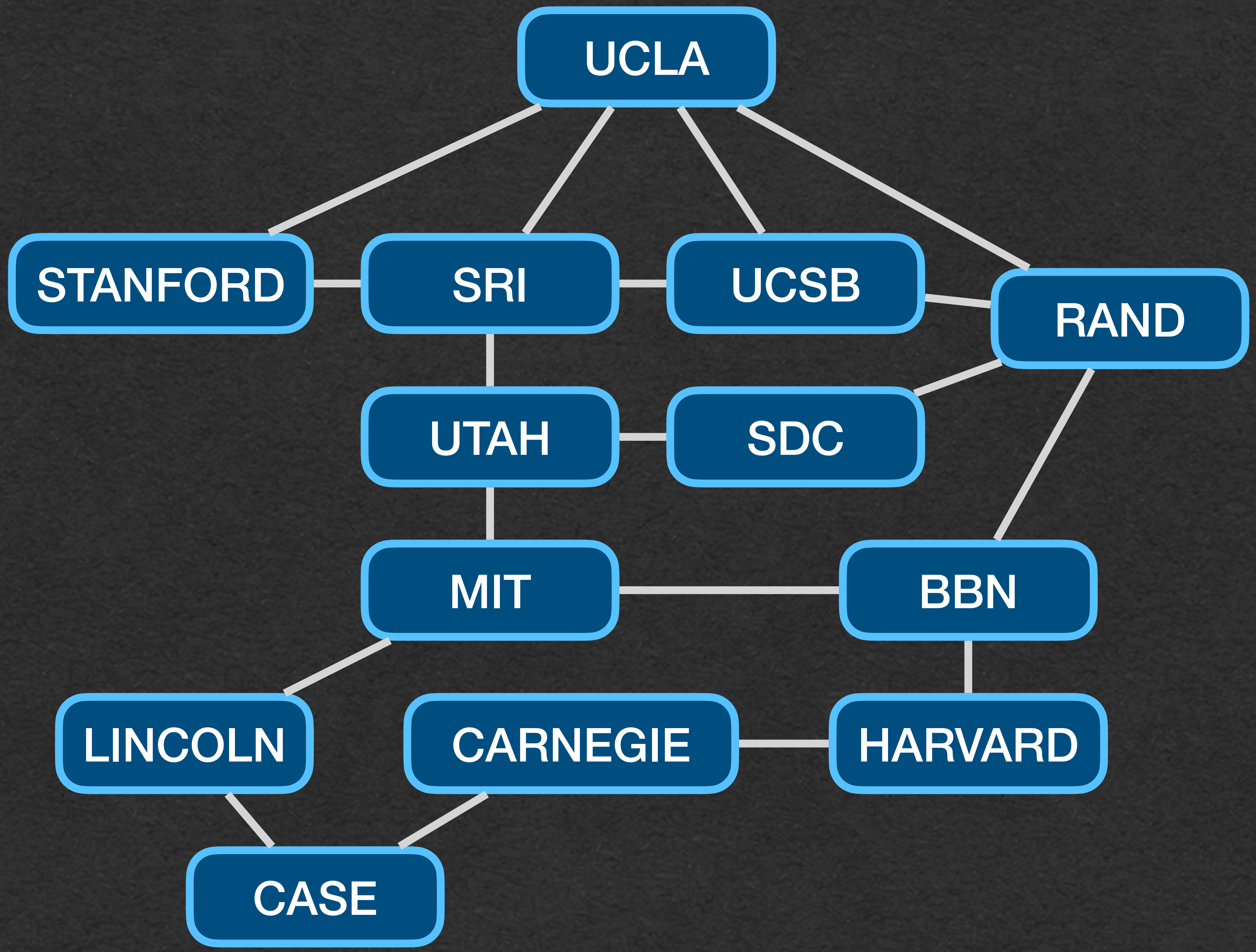
- Node: Each data element is stored in a node, similar to linked lists and trees
- Edge: A connection between two nodes





Graphs - Adjacency List

- A map of nodes to all nodes connected to it through an edge
- This is how we'll represent graphs



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Graphs - Adjacency List

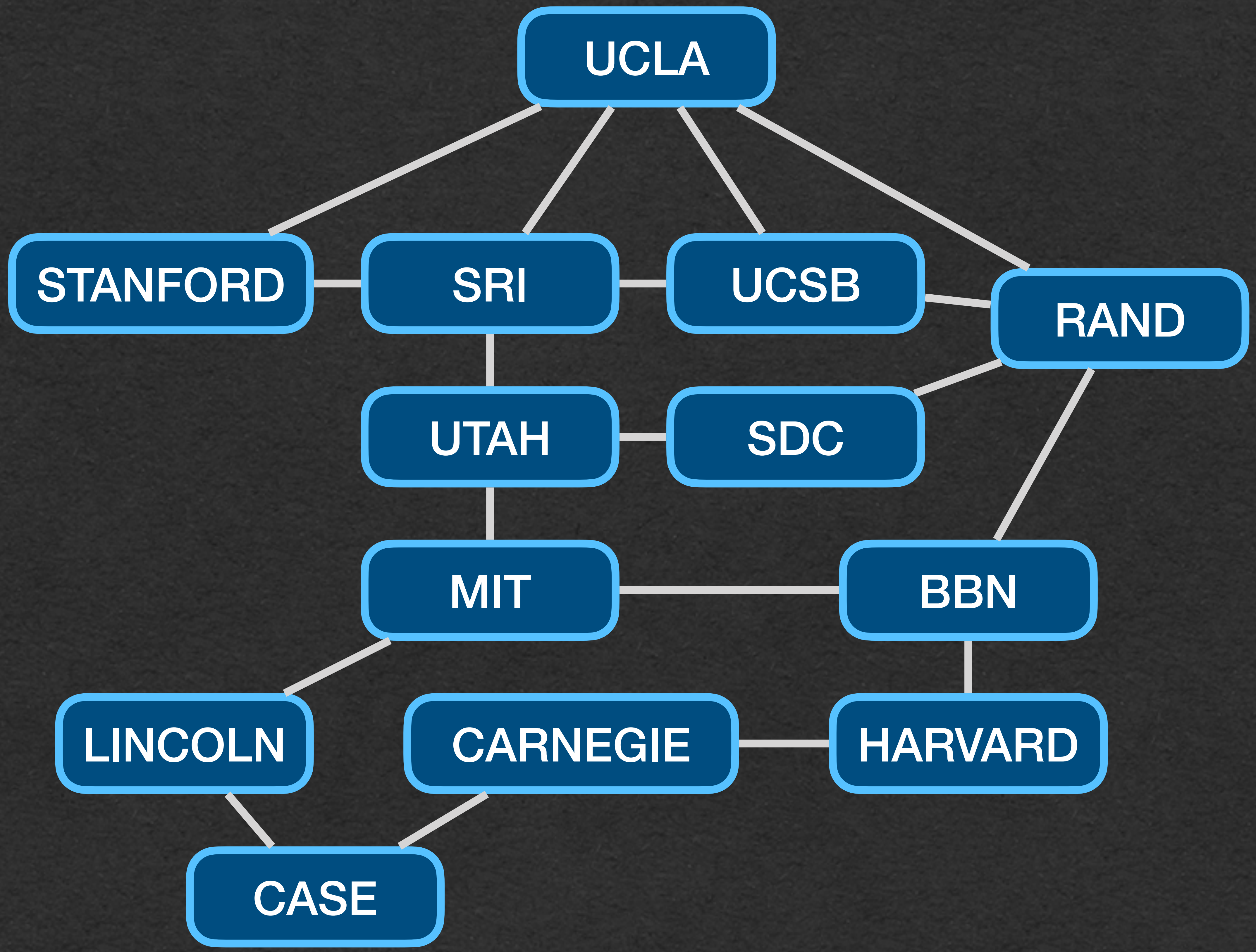
- We use generics in our code so we can create a graph of any type

```
public class Graph<N> {

    private HashMap<N, ArrayList<N>> adjacencyList;

    public Graph() {
        this.adjacencyList = new HashMap<>();
    }
    public void addEdge(N from, N to) {
        this.addNode(from);
        this.addNode(to);
        this.adjacencyList.get(from).add(to);
    }
    public void addBidirectionalEdge(N node1, N node2) {
        this.addNode(node1);
        this.addNode(node2);
        this.adjacencyList.get(node1).add(node2);
        this.adjacencyList.get(node2).add(node1);
    }
    private void addNode(N a) {
        if (!this.adjacencyList.containsKey(a)) {
            this.adjacencyList.put(a, new ArrayList<>());
        }
    }
}
```

```
public static void example(){
    Graph<String> graph = new Graph<>();
    graph.addBidirectionalEdge("UCLA","STANFORD");
    graph.addBidirectionalEdge("UCLA","SRI");
    graph.addBidirectionalEdge("UCLA","UCSB");
    graph.addBidirectionalEdge("UCLA","RAND");
    graph.addBidirectionalEdge("STANFORD","SRI");
    graph.addBidirectionalEdge("SRI","UCSB");
    graph.addBidirectionalEdge("UCSB","RAND");
    graph.addBidirectionalEdge("SRI","UTAH");
    graph.addBidirectionalEdge("RAND","SDC");
    graph.addBidirectionalEdge("UTAH","SDC");
    graph.addBidirectionalEdge("UTAH","MIT");
    graph.addBidirectionalEdge("RAND","BBN");
    graph.addBidirectionalEdge("MIT","BBN");
    graph.addBidirectionalEdge("MIT","LINCOLN");
    graph.addBidirectionalEdge("LINCOLN","CASE");
    graph.addBidirectionalEdge("CASE","CARNEGIE");
    graph.addBidirectionalEdge("CARNEGIE","HARVARD");
    graph.addBidirectionalEdge("HARVARD","BBN");
}
```

Paths

- A path is a sequence of nodes where each pair of adjacent nodes are connected by an edge
- ["UCLA", "SRI", "UTAH", "MIT", "BBN", "RAND"] is a path in this graph
- ["SRI", "UTAH", "BBN"] is not a path since UTAH and BBN are not connected by an edge



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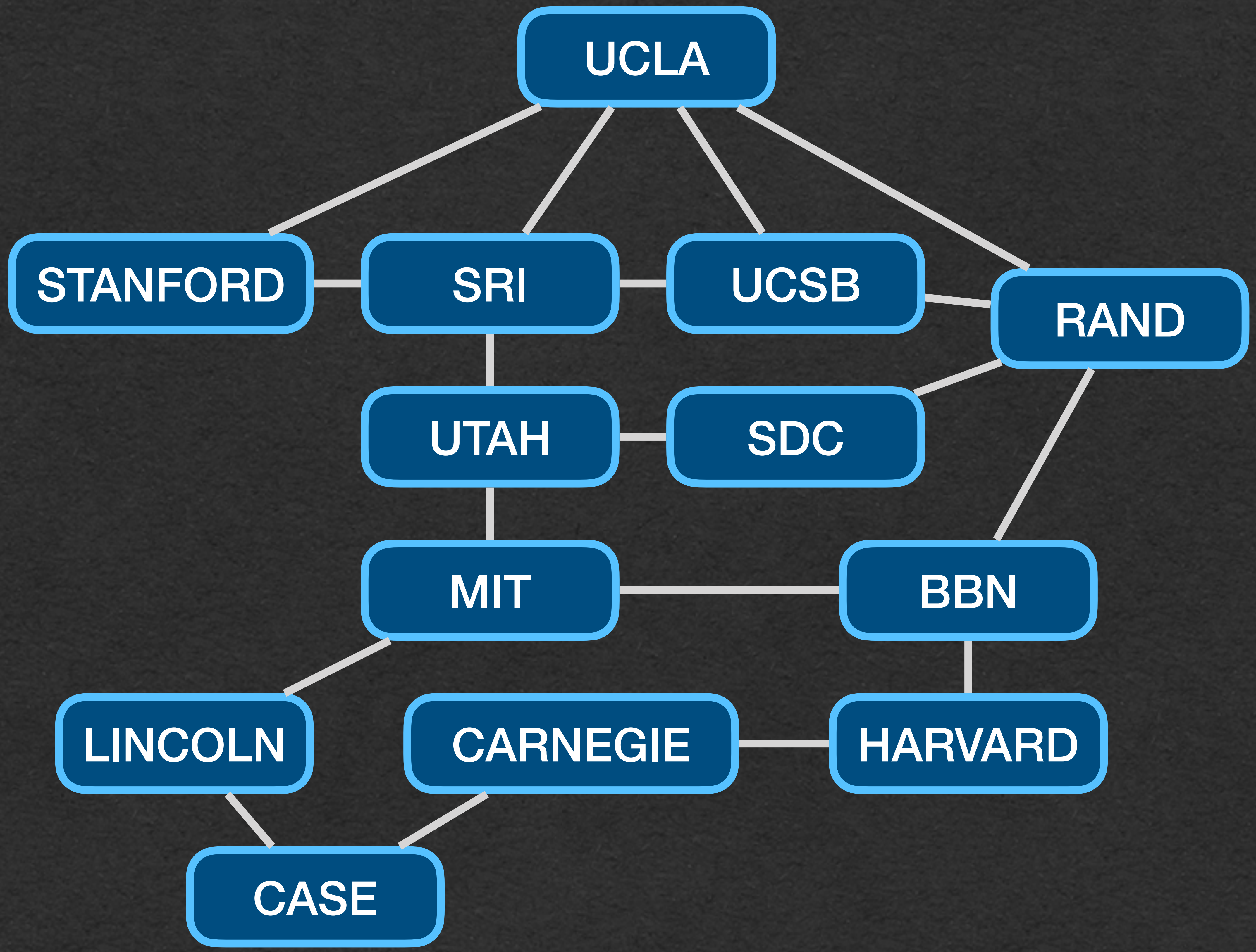
Breadth-First Search (BFS)

Connected Component

- This graph is connected
  - There exists a path between any 2 nodes in the graph



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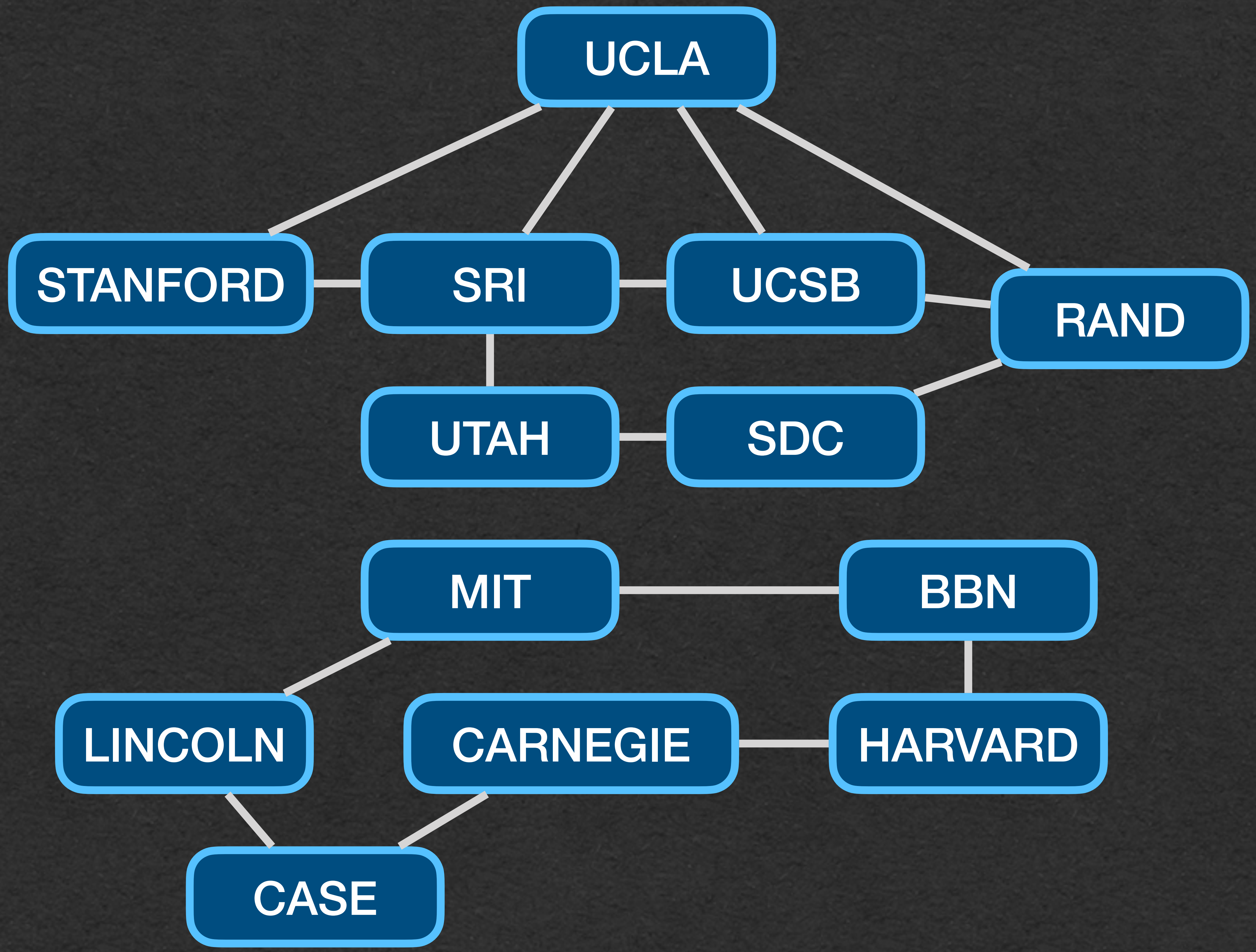
Connected Component

- What if a few connections are broken?
  - How can we tell if two nodes are connected?



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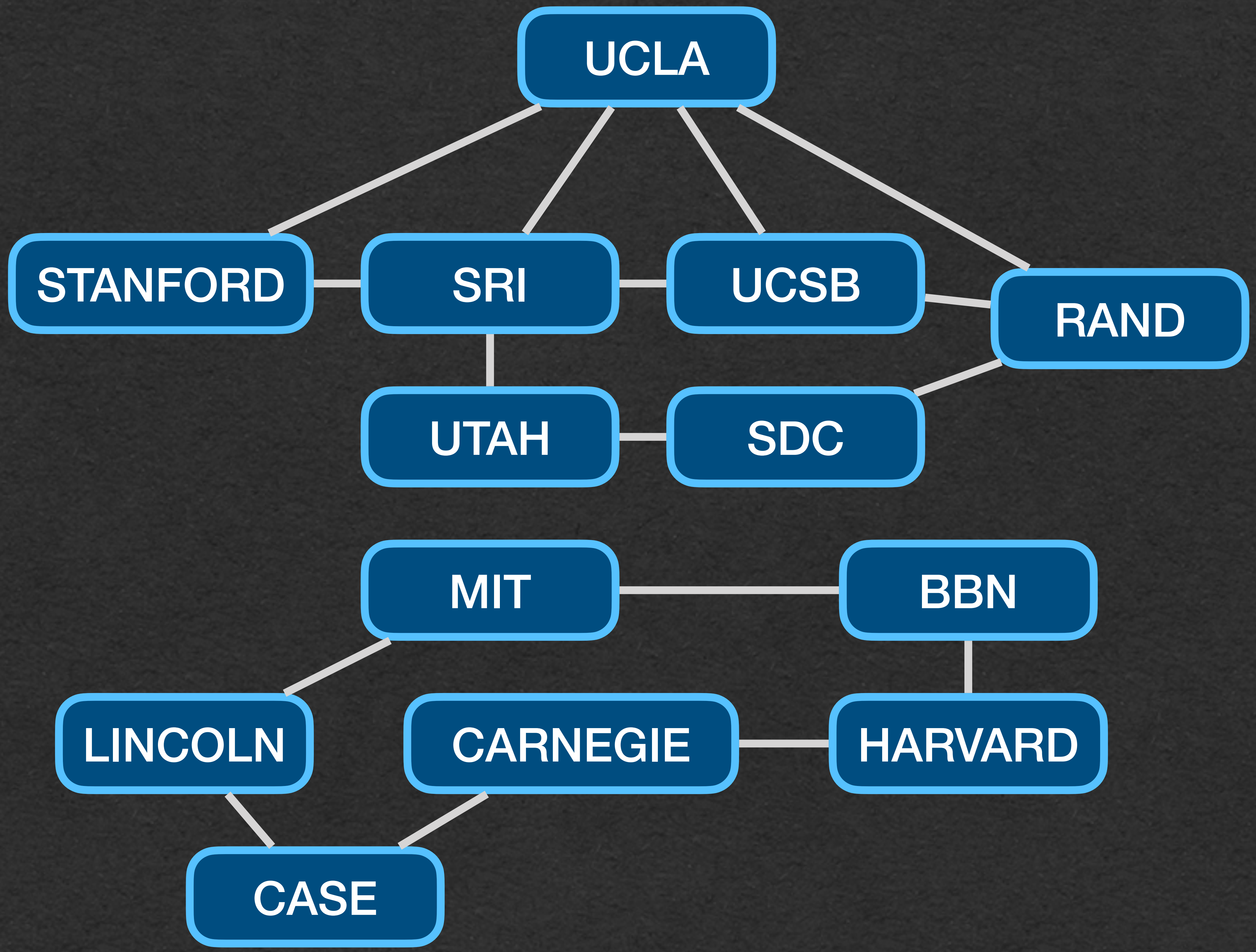
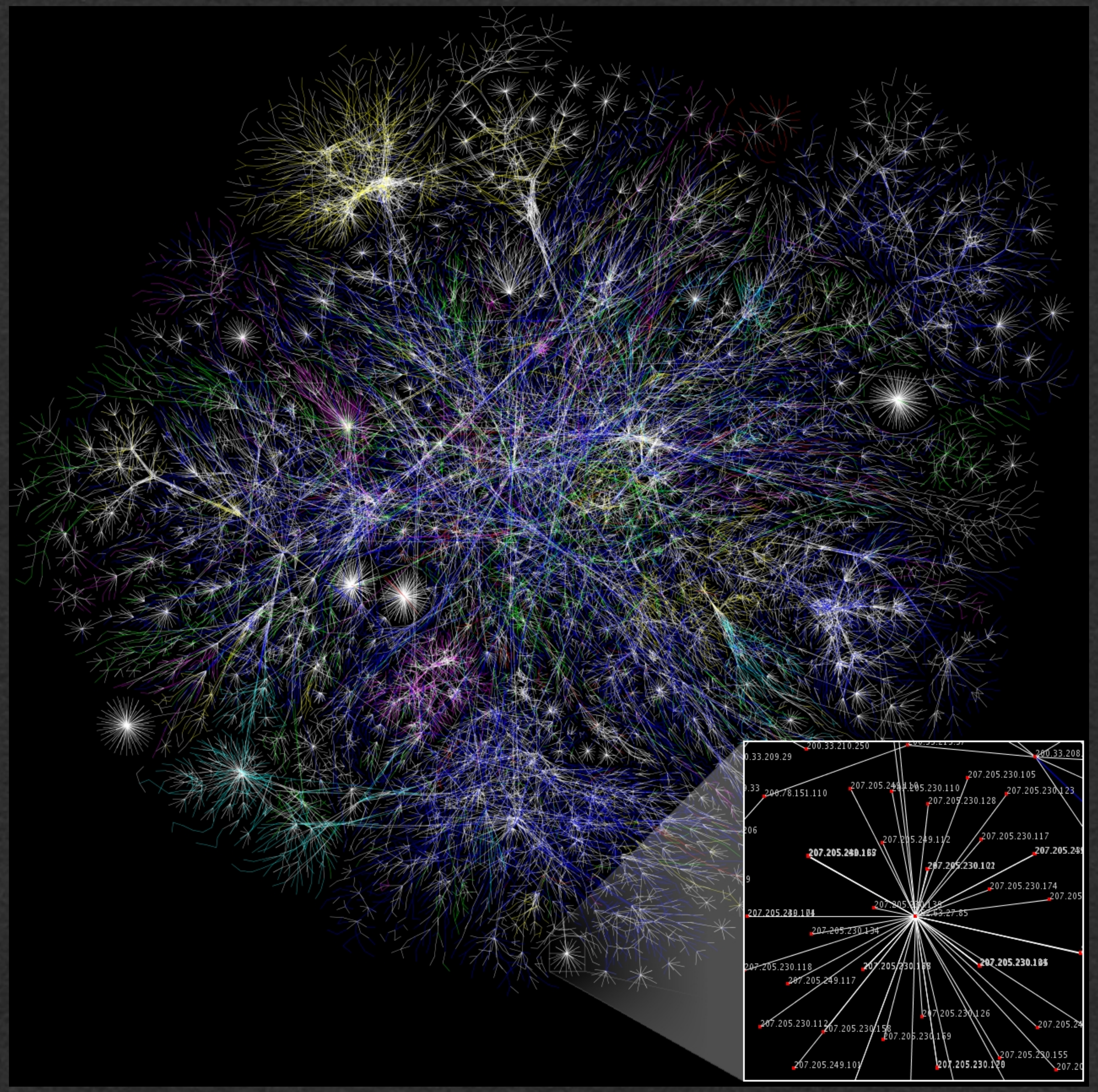
Connected Component

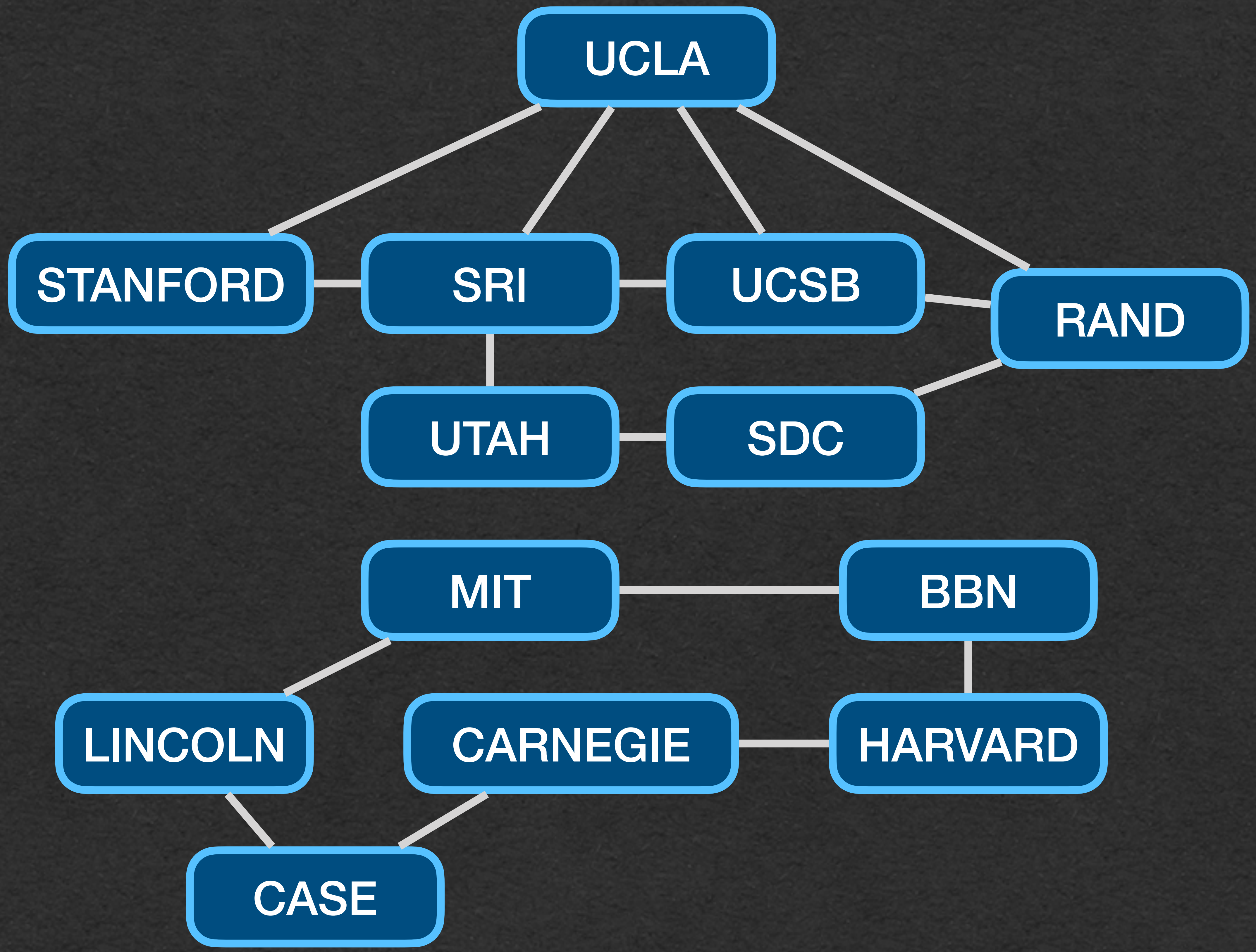
- We could verify manually for this graph
- But the Internet has gotten a *little* bigger over time
- Need to code an algorithm to solve this for us





BFS

- The Algorithm: Breadth-First Search (BFS)
  - Choose a starting node
  - Continuously explore connected nodes



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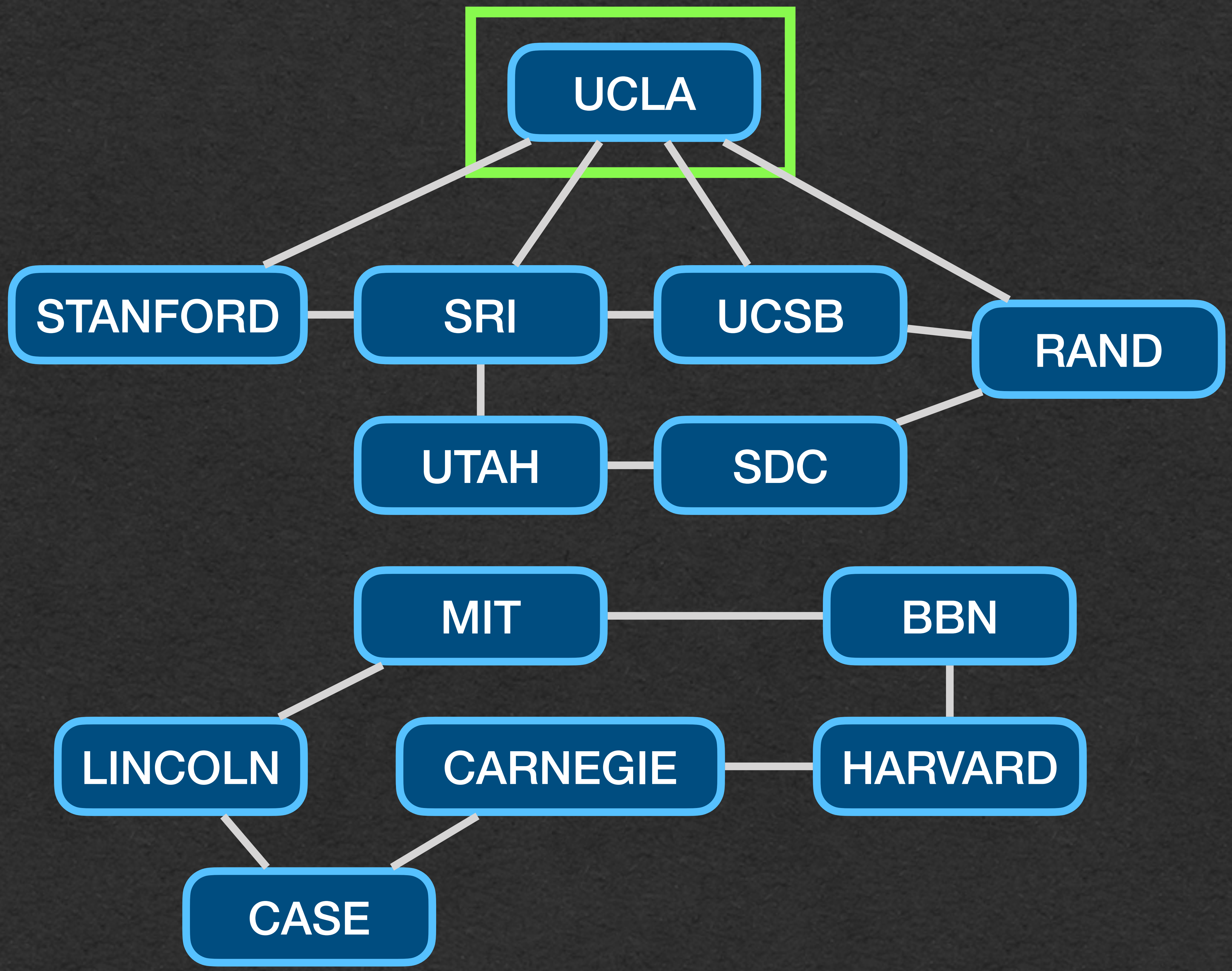
BFS

- Choose a starting node



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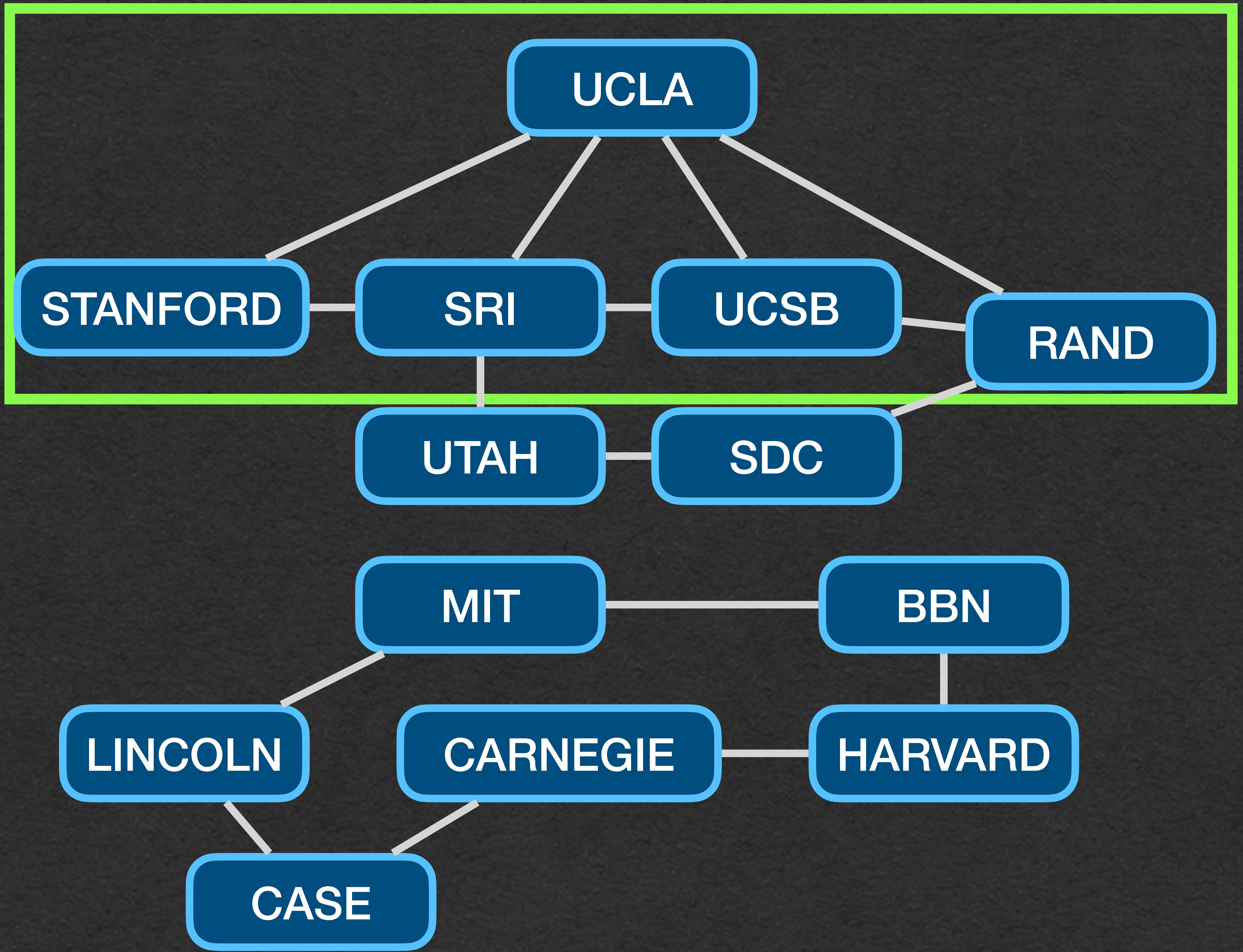
BFS

- Explore all nodes connected to the striating node



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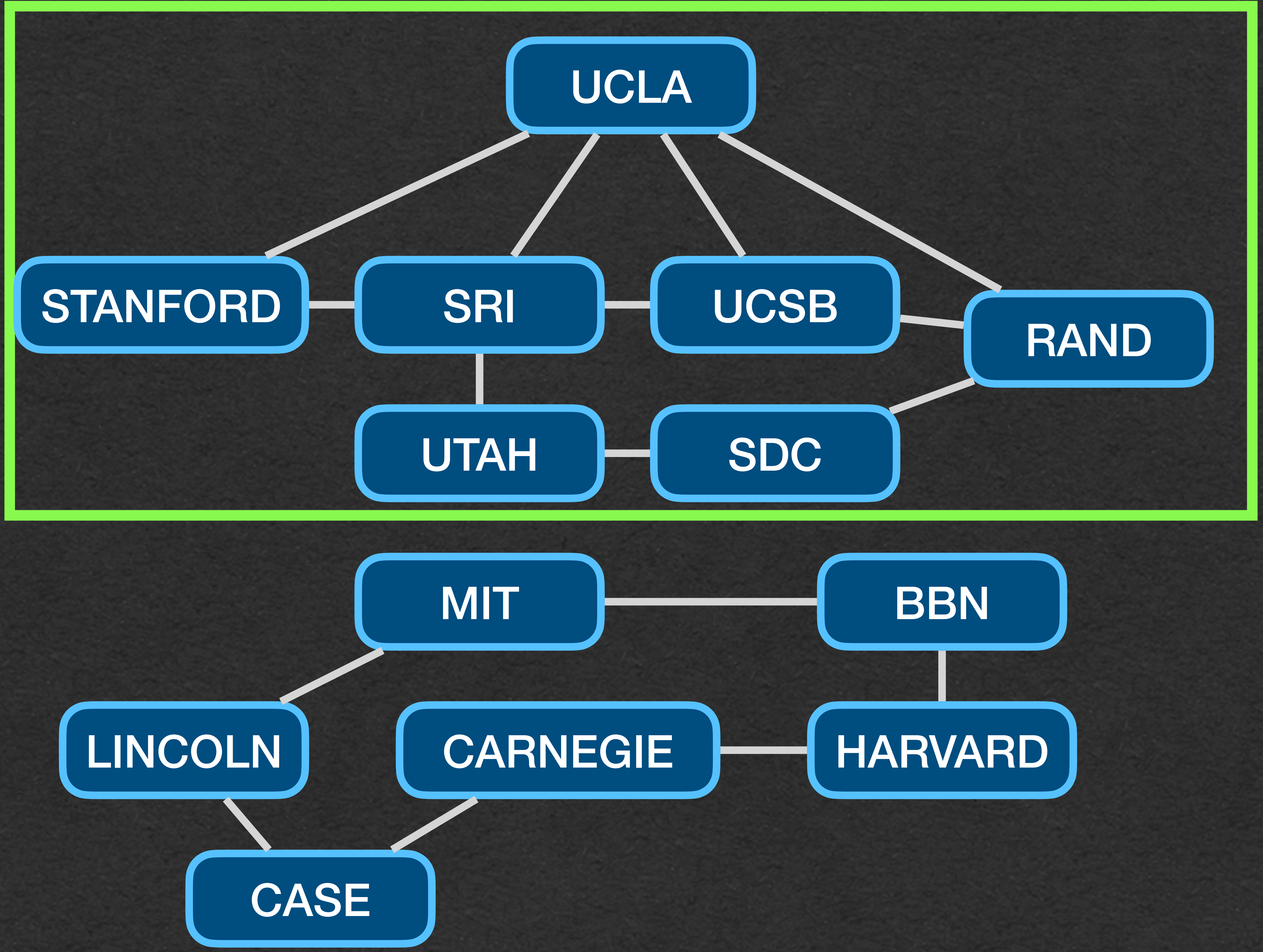
BFS

- Repeatedly explore nodes that were visited in the last round



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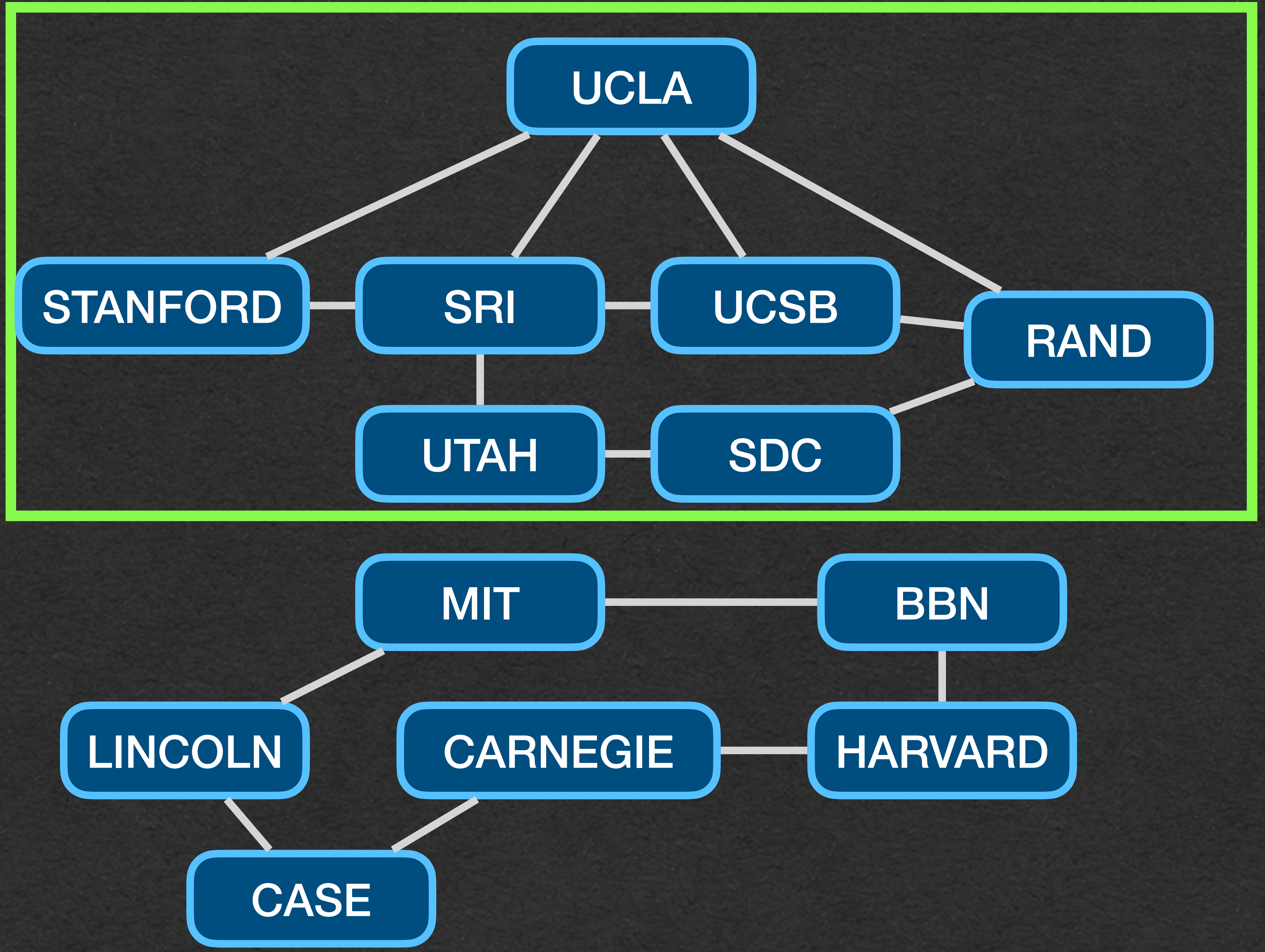
BFS

- Repeat until no new nodes are added
- Never visit a node twice



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BFS

- Use a queue to track the order of nodes to visit
- Start with starting node in the queue
- When visiting a node, add all unexplored neighbors to the queue
- Visit neighbors of the node at the front of the queue until the queue is empty

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BFS

- More BFS details to come

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Connectivity

- If you start at nodeA and explore nodeB during the algorithm
  - nodeA and nodeB are connected